RED ROSE SENIOR SECONDARY SCHOOL

CLASS – XI

Mathematics

ASSIGNMENT PROBLEM – A

1. Find the degree measure corresponding to the following radian measures:
   $(i).\left(\frac{2\pi}{15}\right)^c \quad (ii).\left(\frac{1}{4}\right)^c \quad (iii).-2^c \quad (iv).\ \left(\frac{11}{16}\right)^c \quad Ans.24^0;\ 14^019'5'';\ -(114^032'44'');\ 39^022'30''$
2. Express 1.2 rad in degree measure. $Ans.68^043'(37.8)''$
3. Express the following angles in radians:
   $(i).1' \quad (ii).20^0 \quad (iii).\ 135^0 \qquad Ans.\frac{\pi}{10800};\ \frac{\pi}{9};\ \frac{3\pi}{4}$
4. Express $45^020'10''$ in radian measure $(\pi = 3.1415)$ $Ans.0.79\ rad$
5. Express in radians the forth angle of a quadrilateral which has three angles of$46^030'10'', 75^044'45'', 123^09'35''$ respectively. $Ans.\frac{13751}{21600}\pi$
6. Express in radians and also in degrees the angle of a regular polygon of $(i).40$ sides and $(ii).n$ sides.
   $Ans.\frac{19}{20}\pi, 171^0;\ \left(\frac{n-2}{n}\right)\pi\ rad,\ \left(\frac{n-2}{n}\right)180^0$
7. Find the angle between the minute hand and the hour hand of a clock when the time is 7:20 AM.
   $Ans.100^0$
8. Find in degrees and radians the angle between the hour hand and the minute hand of a clock at half past three. $Ans.75^0;\ \frac{5\pi}{12}\ rad$
9. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15^0. $Ans.\frac{5\pi}{12}$ cm
10. In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord. $Ans.\frac{20\pi}{3}$ cm
11. If arcs of same length in two circles subtend angles of 60^0 and 75^0 at their centers, find the ratios of their radii. $Ans.\ r_1 : r_2 = 5 : 4$
12. The perimeter of a certain sector of a circle is equal to the length of the arc of the semi-circle having the same radius, express the angle of the sector in degrees, minutes and seconds. $Ans.65^027'16''$
13. The angles of a triangle are in A.P. The number of grades in the least, is to the number of radian in the greatest as 40:π. Find the angles in degrees. $Ans.20^0, 60^0\ and\ 100^0$
14. The length of a pendulum is 8 m while the pendulum swings through 1.5 rad, find the length of the arc through which the tip of the pendulum passes. $Ans.12\ cm$
15. The minute hand of a clock is 15 cm long. How does for the tip of the hand move during 40 minutes?
    $Ans.62.8\ cm$
16. A central angle of circle of radius 50 cm intercepts an arc of 10 cm. Express the central angle θ in radians and in degrees. $Ans.\frac{1}{5}\ rad;\ 11^027'17''$
17. A circular wire of radius 2.5 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 1m 29 cm. Find in degrees the angle which is subtended at the centre of the loop.
    $Ans.6.98^0$
18. The moon's distance from the earth is 360000 km and its diameter subtends an angle of $31'$ at the eye of the observer. Find the diameter of the moon. $Ans.3247\frac{13}{21}$

19. If the angular diameter of the moon is $30'$, how for from the eye a coin of diameter 2.2 cm can be kept to hide the moon? $Ans. 252\ cm$
20. A railway train is travelling on a curve of 750 m radius at the rate of 30 km/h, through what angle has it turned in 10 seconds. $Ans. \frac{1}{9} rad$
21. A horse is tethered to a stake by a rope 810 cm long. If the horse moves along the circumference of a circle always keeping the rope tight, find how far it will have gone when the rope has traced out an angle of 70^0. $Ans. 990\ cm$
22. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88 m when it has traced out 72^0 at the center, find the length of the rope. $Ans. 70\ m$
23. The area of the sector is 5.024 cm^2 and its angle is 36^0. Find the radius. $Ans. 4\ cm$
24. Two circles whose centers are A and B tough externally at C. A common tangent touches the circles at P and Q respectively. AP = 8 m and BQ = 2 m. Denoting the angle PAB by α radians, show that the area of the figure bounded by PQ and minor arcs PC and QC is $(40 - 30\alpha - 2\pi)\ m^2$.
25. AB is a diameter of a circle whose centre is O. P is a point on the circumference such that the chord AP = 8 cm and the chord BP = 6 cm. calculate
    a) The values in radians, of the angles PAB and POB. $Ans. \angle PAB = 0.644\ rad;\ \angle POB = 1.29\ rad$
    b) The area of the sector bounded by OP, OB and the minor arc PB. $Ans. 16.1\ cm^2$

Trigonometric function

ASSIGNMENT PROBLEM – B

1. If $\sec\theta = -\frac{13}{12}$ and θ lies in the second quadrant, find the values of all the other five trigonometric functions. $Ans. \sin\theta = \frac{5}{13};\ \cos\theta = -\frac{12}{5};\ \tan\theta = -\frac{5}{12} \cot\theta = -\frac{12}{5}; \operatorname{cosec}\theta = \frac{13}{5}$
2. Find the values of :
    i. $\sin\left(\frac{25\pi}{3}\right)$ $Ans. \frac{\sqrt{3}}{2}$
    ii. $\tan\left(-\frac{16\pi}{3}\right)$ $Ans. -\sqrt{3}$
    iii. $\cot\left(\frac{29\pi}{4}\right)$ $Ans. 1$
    iv. $\operatorname{cosec}\left(\frac{-33\pi}{4}\right)$ $Ans. -\sqrt{2}$
    v. $\cos(15\pi)$ $Ans. -1$
    vi. $\operatorname{cosec}(-1110^0)$ $Ans. -2$
    vii. $\cot(-600^0)$ $Ans. -\frac{1}{\sqrt{3}}$
    viii. $\tan\left(\frac{5\pi}{4}\right)$ $Ans. 1$
3. If $\cos\theta = \frac{-\sqrt{15}}{4}$ and $\frac{\pi}{2} < \theta < \pi$, find the value of $\sin\theta$. $Ans. \frac{1}{4}$

4. If $\cos\theta = -\frac{1}{2}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of $4\tan^2\theta - 3\operatorname{cosec}^2\theta$. $Ans. 8$
5. If $\sec\theta = \sqrt{2}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find the value of $\frac{1+\tan\theta+\operatorname{cosec}\theta}{1+\cot\theta-\operatorname{cosec}\theta}$. $Ans. -1$
6. Prove that : $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \begin{cases} \operatorname{cosec}\theta + \cot\theta, & if\ 0 < \theta < \pi \\ -\operatorname{cosec}\theta - \cot\theta, & if\ \pi < \theta < 2\pi \end{cases}$
7. Prove that : $\cos 510^0 \cos 330^0 + \sin 390^0 \cos 120^0 = -1$.
8. Prove that : $i.\ \tan 720^0 - \cos 270^0 - \sin 150^0 \cos 120^0 = \frac{1}{4}$
   $ii.\ \sin 780^0 \sin 480^0 + \cos 120^0 \sin 150^0 = \frac{1}{2}$
   $iii.\ \sin 600^0 \cos 390^0 + \cos 480^0 \sin 150^0 = -1$
9. Prove that : $i.\ \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$
   $ii.\ \frac{\sin(180^0+\theta)\cos(90^0+\theta)\tan(270^0-\theta)\cot(360^0-\theta)}{\sin(360^0-\theta)\cos(360^0+\theta)\operatorname{cosec}(-\theta)\sin(270^0+\theta)} = 1$
   $iii.\ 3\left\{\sin^4\left(\frac{3\pi}{2}-\theta\right) + \sin^4(3\pi+\theta)\right\} - 2\left\{\sin^6\left(\frac{\pi}{2}+\theta\right) + \sin^6(5\pi-\theta)\right\} = 1$
   $iv.\ \left\{1+\cot\theta-\sec\left(\frac{\pi}{2}+\theta\right)\right\}\left\{1+\cot\theta+\sec\left(\frac{\pi}{2}+\theta\right)\right\} = 2\cot\theta$
   $v.\ \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$
10. Show that $\sin^2 5^0 + \sin^2 10^0 + \sin^2 15^0 \ldots\ldots\ldots\ldots + \sin^2 90^0 = 9\frac{1}{2}$
11. Show that $\sec^2\theta + \operatorname{cosec}^2\theta \geq 4$.
12. Show that $\sin^2\theta + \operatorname{cosec}^2\theta \geq 2$.
13. If x is any real number then show that $\cos\theta$ cannot be equal to $x + \frac{1}{x}$.
14. Can $6\sin^2\theta - 7\sin\theta + 2 = 0$ for any real value of θ. $Ans. yes$
15. Show that $\sin^2\theta = \frac{x^2+y^2}{2xy}$ is possible for real values of x and y only when $x = y \neq 0$.

Trigonometric Ratios of Compound Angles

ASSIGNMENT PROBLEM – C

1. If $\cos A = \frac{4}{5}, \cos B = \frac{12}{13}, \frac{3\pi}{2} < A, B < 2\pi$, find the values of the $\cos(A+B)$ and $\sin(A-B)$.
   $Ans. \frac{33}{65}; \frac{-16}{65}$
2. If $\cot\alpha = \frac{1}{2}, \sec\beta = -\frac{5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. find the value of $\tan(\alpha+\beta)$. State the quadrant in which $\alpha+\beta$ terminates. $Ans. \frac{2}{11}; I\ quadrant$
3. Find the values of the following :
   $i.\sin 75^0$ $ii.\ \cos 75^0$ $iii.\sin 15^0$ $iv.\cos 15^0$
   $Ans. \frac{\sqrt{3}+1}{2\sqrt{2}}; \frac{\sqrt{3}-1}{2\sqrt{2}}; \frac{\sqrt{3}-1}{2\sqrt{2}}; \frac{\sqrt{3}+1}{2\sqrt{2}}$
4. Prove that : $\cos\left(\frac{\pi}{4}-A\right)\cos\left(\frac{\pi}{4}-B\right) - \sin\left(\frac{\pi}{4}-A\right)\sin\left(\frac{\pi}{4}-B\right) = \sin(A+B)$
5. Prove that : $\frac{\sin(x+y)}{\sin(x-y)} = \frac{\tan x + \tan y}{\tan x - \tan y}$.

6. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, prove that $\cot(A - B) = \frac{1}{x} + \frac{1}{y}$.
7. If α and β are acute angle such that $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, prove that $\alpha + \beta = \frac{\pi}{4}$.
8. Prove that : $i.$ $\tan 3A \tan 2A \tan A = \tan 3A - \tan 2A - \tan A$
   $ii.$ $\cot A \cot 2A - \cot 2A \cot 3A - \cot 3A \cot A = 1$
9. Prove that : $\frac{\cos^2 33^0 - \cos^2 57^0}{\sin^2\frac{21^0}{2} - \sin^2\frac{69^0}{2}} = -\sqrt{2}$
10. If $3\tan A \,.\tan B = 1$, prove that $2\cos(A + B) = \cos(A - B)$
11. If $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha) = -\frac{3}{2}$, prove that $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 1$
12. If $\sin B = 3\sin(2A + B)$, prove that $2\tan A + \tan(A + B) = 0$
13. If $\cos(\alpha + \beta)\sin(\gamma + \delta) = \cos(\alpha - \beta)\sin(\gamma - \delta)$, prove that $\cot\alpha \cot\beta \cot\gamma = \cot\delta$.
14. Prove that : $\frac{\sin(x+\theta)}{\sin(x+\emptyset)} = \cos(\theta - \emptyset) + \cot(x + \emptyset)\sin(\theta - \emptyset)$.
15. If $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$, prove that $\tan 2\alpha = \frac{56}{33}$
16. Prove that : $\tan 70^0 = \tan 20^0 + 2\tan 50^0$
17. If $\tan(\alpha + \theta) = n\tan(\alpha - \theta)$, show that : $(n+1)\sin 2\theta = (n-1)\sin 2\alpha$.
18. If $a\tan\alpha + b\tan\beta = (a+b)\tan\left(\frac{\alpha+\beta}{2}\right)$, where $\alpha \neq \beta$, prove that $a\cos\beta = b\cos\alpha$.
19. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, show that : $i.\cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$, $ii.\sin(\alpha + \beta) = \frac{2ab}{a^2 - b^2}$
20. If α and β are the solutions of the equation $a\tan\theta + b\sec\theta = c$, then show that $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$
21. Prove that : $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = \frac{1}{2}(\tan 27x - \tan x)$.
22. If angle θ is divided into two parts such that the tangents of one part is α times the tangent of the other, and $\emptyset$ is their difference, then show that $\sin\theta = \frac{\alpha+1}{\alpha-1}\sin\emptyset$.
23. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.
24. If $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$, prove that $1 + \cot\alpha\tan\beta = 0$.
25. If $\tan\alpha = x + 1, \tan\beta = x - 1$, show that $2\cot(\alpha - \beta) = x^2$.